Student's *t* Distribution

Lecture 35
Section 10.2

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Outline

Introduction

- We will familiarize ourselves with the *t* distribution.
- Then we will see how to use it to test a hypothesis concerning μ when σ is not known.
- We will learn how to perform the *z*-test and the *t*-test on the TI-83.

When to Use Z

- Use Z whenever
  - The sample size is large ($n \geq 30$), or
  - The population is normal and σ is known.

When to use t

- Use t when
  - The population is normal, and
  - σ is not known, and (optionally)
  - The sample size is small.

When to Give Up

- Give up when
  - The population is not normal, and
  - The sample size is small ($n < 30$).

TI-83 - Student's *t* Distribution

- The TI-83 will find probabilities for the *t* distribution (but not percentiles, in general).
  - Press `DISTR`.
  - Select `tcdf` and press `ENTER`. `tcdf(` appears in the display.
  - Enter the lower endpoint.
  - Enter the upper endpoint.
  - Enter the number of degrees of freedom ($n - 1$).
  - Press `ENTER`. The result is the probability.

Width of *t*-Distribution

- Compute
  - `tcdf(-1.0, 1.0, 10).`
  - `tcdf(-2.0, 2.0, 10).`
  - `tcdf(-3.0, 3.0, 10).`
- We see that the values are smaller than 68%, 95%, and 99.7%.
- What does this tell us?

Upper Tails of t-Distribution

- Compute
  - `tcdf(1.960,E99,2).`
  - `tcdf(1.960,E99,10).`
  - `tcdf(1.960,E99,30).`
  - `tcdf(1.960,E99,100).`
  - `normalcdf(1.960,E99).`
- What does this tell us?

Hypothesis Testing with t

- Resume the Example 10.1, p. 616.
- Recall the first two steps:
- Step 1: State the hypotheses.

$$H_0 : \mu = 15$$

$$H_1 : \mu < 15$$

- Step 2: State the value of α: $\alpha = 0.05$.
- Now we are ready to continue with Step 3.

Hypothesis Testing with t

- Step 3: Write the formula for the test statistic.
  - The test statistic is now

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

- Step 4: Compute the value of the test statistic.
  - Use the sample data to compute $\overline{x}$, and s.
  - Then compute t from the formula.

Hypothesis Testing with t

- Step 5: Find the *p*-value.
  - Use `tcdf` on the TI-83.
- Step 6: Make the decision regarding H_0.
- Step 7: State the conclusion about the carbon-monoxide content of cigarettes.

Example Summary

- Summary of the seven steps.
  1. μ represents the average carbon-monoxide content of a cigarette today.
     $H_0 : \mu = 15$ mg
     $H_1 : \mu = 15$ mg
  2. $\alpha = 0.05$.
  3. $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$
  4. $t = \dfrac{12.528 - 15}{4.74/\sqrt{25}} = \dfrac{-2.472}{0.948} = -2.608$
  5. p-value $=$ `tcdf(-E99,-2.608,24)` $= 0.007711$.
  6. Reject H_0.
  7. The carbon-monoxide content of cigarettes today is less than 15 mg.

TI-83 - Hypothesis Testing When σ is Unknown

- We can perform a *t* test on the TI-83.
  - Press `STAT`.
  - Select `TESTS`.
  - Select `T-Test`. A window appears requesting information.
  - Choose `Data` or `Stats`.

TI-83 - Hypothesis Testing When σ is Unknown

- Assuming we selected `Stats`,
  - Enter μ_0.
  - Enter $\overline{x}$.
  - Enter s. (Remember, σ is unknown.)
  - Enter n.
  - Select the alternative hypothesis and press `ENTER`.
  - Select `Calculate` and press `ENTER`.

TI-83 - Hypothesis Testing When σ is Unknown

- A window appears with the following information.
  - The title `T-Test`
  - The alternative hypothesis.
  - The value of the test statistic t.
  - The p-value.
  - The sample mean.
  - The sample standard deviation.
  - The sample size.

Example

- Re-do Example 10.1, p. 616, on the TI-83 under the assumption that σ is unknown.
  - Work it once using `Stats`.
  - Work it again using `Data`.

Example

- Work Example 10.3 on page 628.
- Course ratings given by the 20 females:

5.2	6.8	7.0	7.8	6.4	6.5	7.2	8.4
6.7	9.2	5.9	6.3	7.8	7.7	8.2	6.9
8.1	8.0	7.5	7.0				

- Construct a QQ plot to see whether normality is reasonable.
- Is there sufficient evidence, at the 5% level of significance, to conclude that the average of the females' scores is less than 7.5?

Summary

Summary

- The TI-83 will perform the z-test and the t-test.
- Actually, it will perform the calculations in steps 4 and 5 of those tests.
- The TI-83 will also find probabilities for the t distribution.
- The TI-83 will not read the problem and decide which test to use.
- Use the t-test if
  - The population is normal, and
  - σ is unknown.